Module 4:

Making Arguments

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Module Learning Outcomes	
By the end of this module students will be able to:	

1. Create claims in an engineering context. [CLO 3, 5]
2. Structure an argumentative paragraph using precise language. [CLO 2, 3, 5]
3. Justify a claim with the most relevant evidence. [CLO 5]
4. Make judgements about the quality of evidence and justification. [CLO 3, 4, 5]
5. Evaluate a source using the CRAAP test. [CLO 6]

Resources (Bank)	
Item	**Description of how to be used**
Exercise 1 – Correct Order of Sentences	Used to draw attention to logical order of sentences at the beginning of the workshop
Exercise 2 – Ordering Sentences into Paragraphs	Used to review logical organization of sentences into a meaningful paragraph
Research Article (Rohrer, Dedrick, Burgess, 2014)	Used to help students identify claim(s) and evidence and how they can be placed in a piece of writing
Supplemental Exercise – Putting Words in the Correct Order	The purpose of this exercise is to draw student attention to sentence structure. This is particularly useful if students are having difficulty constructing a syntactically correct sentence.

Face to Face Workshop Plan	
Description of Workshop	This workshop provides students an opportunity to practice recognizing the elements of an argument, which is one of the most common genres used in engineering communication. It also provides them with supervised practice in organizing sentences within a paragraph in order to produce a logical argument, inductively or deductively.

Time for Completion	90 minutes
Materials	Exercise Sheet 1 - Correct Order of Sentences – one per student Exercise Sheet 2 - Ordering Sentences into Paragraphs - one per student Exercise 1 & 2 answer sheets Copies of research article - one per student Coloured pens or highlighters
Workshop Preparation Instructions	The facilitator should read the article carefully, identifying the various claims, qualifiers, evidence, and justifications used throughout. The facilitator should also identify specific paragraphs to be used by pairs of students where the argument structure is particularly salient.
Procedure	**Step 1: Constructing a simple argument** (~5 minutes) *Facilitator Notes:* The following exercise focuses on sentence order to help students identify a simple argument structure—background, claim, evidence, justification or linking the evidence with the claim. The exercise also acts as a self-diagnostic in that it will show students where they may experience difficulty or have control over paragraph structure. • Give students the Exercise Sheet 1: Correct Order of Sentences and allow them five minutes to complete the exercise. • Give or display answer key and answer questions about alternative constructions, if there are any. **Step 2: Recognizing a claim** (~15 minutes) *Facilitator Notes:* Students are now primed to notice how sentences work together to construct a concise argument. Concise sentences are used in abstracts to state the article's claim(s) and the most important evidence. • Give students a copy of the Rohrer, Dedrick, Burgess (2014) article and instruct them to read the abstract (and only the abstract).

- Students should underline what they believe to be the main claim of the article as expressed in the abstract in one colour.
- Students should underline main pieces of evidence found in the abstract in a different colour.

The following is the abstract with the claims and evidence highlighted.

> Most mathematics assignments consist of a group of problems requiring the same strategy. For example, a lesson on the quadratic formula is typically followed by a block of problems requiring students to use that formula, which means that students know the appropriate strategy before they read each problem. In an alternative approach, different kinds of problems appear in an interleaved order, which requires students to choose the strategy on the basis of the problem itself. In the classroom-based experiment reported here, grade 7 students (n = 140) received blocked or interleaved practice over a nine-week period, followed two weeks later by an unannounced test. The mean test scores were greater for material learned by interleaved practice rather than by blocked practice (72 % vs. 38 %, d = 1.05). This interleaving effect was observed even though the different kinds of problems were superficially dissimilar from each other, whereas previous interleaved mathematics studies had required students to learn nearly identical kinds of problems. We conclude that interleaving improves mathematics learning not only by improving discrimination between different kinds of problems, but also by strengthening the association between each kind of problem and its corresponding strategy.

(Rohrer, D., Dedrick, D. F. & Burgess, K., 2014. “The benefit of interleaved mathematics practice is not limited to superficially similar kinds of problems”. Psychon Bull Rev 21:1323–1330 DOI 10.3758/s13423-014-0588-3)

Step 3: Exercise 2 Constructing a Paragraph (~10-15 minutes)

	Facilitator Notes: The following is an exercise meant to get the students thinking about how a paragraph is organized to make an argument. It uses something easy to understand but still demands that there needs to be logical connections. The facilitator can point out to the students that the same principles of organization apply to the paragraphs they are about to read in the article in the next step. Distribute Exercise Sheet 2 - Ordering Sentences into Paragraphs. Instruct students to read the sentences and then re-organize them into logical, linked paragraphs. They can just number the sentences rather than re-write them, although students may find it easier to see the logical connections if they write them out. **Step 4: Reading to Identify Parts of an Argument** ~15 minutes) *Facilitator Notes:* This exercise is meant to be short. The facilitator should have already identified the paragraphs he or she has chosen to assign to students. Paragraphs in the first two sections or in the discussion section of the article may be most useful. The students are only to identify the parts of the paragraphs they are assigned. Most of the paragraphs in this article are relatively short, so should not take that long to read. Students need not understand every word in the paragraph to identify the claim, evidence, etc. The instructor can point out to the students that authors often use words like "The evidence suggests that…. Or "We claim that…." Divide the group into pairs. Assign each pair one paragraph in the article to identify the claims, the evidence, any qualifiers, and the justification. Do not use more than 5 minutes for reading. After the pairs have finished, ask them to share their claims, evidence, justifications, etc. Call attention, in particular, to the use of words such as "because, as a result, even though, etc." as well as order of sentences to show relationships between the ideas. **Step 5: CRAAP Test** (~15 minutes) *Facilitator Notes:* This step is meant to introduce students to one commonly accepted method of evaluating sources used in an argument. Students are asked to choose one of the references

	from the article and do the CRAAP test on it. They are expected to Google authors, publishers, etc. in order to make their best judgments about the quality and appropriateness of the source they choose. • Currency—is the article current? Currency is usually thought of as in the past five years, however, some seminal pieces of research or books written 15 or more years ago may still be current. Standards and regulations change more slowly, so these topics can have a longer period of currency. Something in fields like biomed or artificial intelligence etc., is probably measured in much shorter time periods. • Relevance—is the content of the source relevant to the question or subject being considered? • Authority—does the author (or authors) have authority in the subject? Although Steve Jobs may have been an amazing designer, he probably had little authority when it came to the best way to raise puppies. • Accuracy—how accurate do you judge the information to be? Information supplied by most government sites is usually more accurate than information supplied by a marketing site. Information provided by a standards organization such as Underwriters Laboratory is usually more accurate than information provided by a blogger. • Purpose—For what purpose was the information produced? If the article or information was produced to persuade readers or to convey information, the content may be different. If the purpose is aligned with the person using the information, it is probably more reliable or useful. **Step 6: Writing an Argumentative Paragraph** (~20 minutes) *Facilitator Notes:* Students should have some idea of what interleaving vs blocked practice with mathematical problems is all about. (Note: Interleaving refers to switching from one subject or type of problem to another in a set time. Blocked practice refers to focusing on only one subject or type of problem for an extended period of time.) They are all familiar with "problem sets" and learning new strategies and formulae to use in solving problems.

	Writing a paragraph that makes a claim about one way of learning new formulae, using evidence from the article or their own experience, should not be too challenging for the students. Students need only make one major claim. Based on the work the students have done, ask each student to write one paragraph making an argument about the efficacy of interleaving or blocked study practices. At the end of 20 minutes (or earlier if most students have finished) each student gives his or her paragraph to a partner. The partners mark what they see as the claim, evidence, and justification.
Supplemental Materials	Supplemental Word Order Exercise—The purpose of this exercise is to draw student attention to sentence structure. This is particularly useful if students are having difficulty constructing a syntactically correct sentence.
Assessment	Student-produced paragraphs with identifiable claim, evidence, and justification.

Resources

CORRECT ORDER OF SENTENCES

This worksheet is designed to build your skills in putting sentences in the correct order.

EXERCISE

You will be constructing an email out of the information in the chart below. Each line on the right will be filled in with information from the left.

Items to Move	Move items into this column
I attached the doctor's note.	
Message:	
Thank you.	
Please confirm that the cost of the medicine is covered by the company healthcare plan.	
I was sick yesterday, and therefore I couldn't come to work.	
To: Jennifer Brown, Human Resources	
Date: October 26, 2011	
Subject: Sick leave certificate	
In order to receive sick pay, I need to send in my doctor's note.	
From: Mark Green, Sales	

CORRECT ORDER OF SENTENCES

This worksheet is designed to build your skills in putting sentences in the correct order.

ANSWERS

Items to Move	Move items into this column
I attached the doctor's note.	From: Mark Green, Sales
Message:	To: Jennifer Brown, Human Resources
Thank you.	Date: October 26, 2011
Please confirm that the cost of the medicine is covered by the company healthcare plan.	Subject: Sick leave certificate
I was sick yesterday, and therefore I couldn't come to work.	Message:
To: Jennifer Brown, Human Resources	I was sick yesterday, and therefore I couldn't come to work.
Date: October 26, 2011	In order to receive sick pay, I need to send in my doctor's note.
Subject: Sick leave certificate	I attached the doctor's note.
In order to receive sick pay, I need to send in my doctor's note.	Please confirm that the cost of the medicine is covered by the company healthcare plan.
From: Mark Green, Sales	Thank you.

ORDERING SENTENCES INTO PARAGRAPHS

This worksheet is designed to build your skills in putting sentences in the correct order to form paragraphs.

EXERCISE

Read the following sentences. Arrange and group them in order and into paragraphs. Write the completed paragraphs in the space provided.

- In those days, lunch was served at noon, but dinner was not eaten until late at night.
- In fact, they are two entirely different things.
- Afternoon tea began in the mid-1800s.
- A noblewoman, the Duchess of Bedford, found herself hungry during those long afternoon hours and so she started having a tray of tea with bread and butter served to her in the mid-afternoon.
- Most people think that afternoon tea is synonymous with high tea.
- And although high tea sounds classy, it actually consisted of a full dinner for the common people.
- Soon, she began to invite other ladies to join her.
- High tea, on the other hand, was served around six in the evening.
- Tea was still served, but there would also be meats, fish or eggs, cheese, bread and butter, and cake.
- Without realizing it, the Duchess of Bedford was setting the trend of having afternoon tea for the upper-class women.

Completed Paragraph

ORDERING SENTENCES INTO PARAGRAPHS

ANSWER

Most people think that afternoon tea is synonymous with high tea. In fact, they are two entirely different things.

Afternoon tea began in the mid-1800s. In those days, lunch was served at noon but dinner was not eaten until late at night. A noblewoman, the Duchess of Bedford, found herself hungry during those long afternoon hours and so she started having a tray of tea with bread and butter served to her in the mid-afternoon.

Soon, she began to invite other ladies to join her. Without realizing it, the Duchess of Bedford was setting the trend of having afternoon tea for the upper- class women.

High tea, on the other hand, was served around six in the evening. And although high tea sounds classy, it actually consisted of a full dinner for the common people. Tea was still served, but there would also be meats, fish or eggs, cheese, bread and butter, and cake.

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BRIEF REPORT

The benefit of interleaved mathematics practice is not limited to superficially similar kinds of problems

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Abstract Most mathematics assignments consist of a group of problems requiring the same strategy. For example, a lesson on the quadratic formula is typically followed by a block of problems requiring students to use that formula, which means that students know the appropriate strategy before they read each problem. In an alternative approach, different kinds of problems appear in an interleaved order, which requires students to choose the strategy on the basis of the problem itself. In the classroom-based experiment reported here, grade 7 students (n = 140) received blocked or interleaved practice over a nine-week period, followed two weeks later by an unannounced test. The mean test scores were greater for material learned by interleaved practice rather than by blocked practice (72 % vs. 38 %, d = 1.05). This interleaving effect was observed even though the different kinds of problems were superficially dissimilar from each other, whereas previous interleaved mathematics studies had required students to learn nearly identical kinds of problems. We conclude that interleaving improves mathematics learning not only by improving discrimination between different kinds of problems, but also by strengthening the association between each kind of problem and its corresponding strategy.

Keywords Learning · Mathematics · Interleaved · Spacing · Practice

Learning techniques inspired by research in the laboratory can improve learning in the classroom (for recent reviews, see Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013; Roediger & Pyc, 2012). In the study reported here, a simple intervention designed to improve mathematics learning was assessed in a classroom-based experiment. We first describe the intervention and the relevant research.

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Interleaved practice

The solution of a mathematics problem requires two steps, as is illustrated by the following example:

> A bug flies 48 m east and then flies 14 m north. How far is the bug from where it started?

This problem is solved by using the Pythagorean theorem to find the length of a hypotenuse ($\sqrt{48+14}=50$). In other words, students first choose a strategy (Pythagorean theorem) and then execute the strategy. The term *strategy* is used loosely here to refer to a theorem, formula, concept, or procedure. Learning to choose an appropriate strategy is difficult, partly because the superficial features of a problem do not always point to an obvious strategy (e.g., Chi, Feltovich, & Glaser, 1981; Siegler, 2003). For example, the word problem about the bug does not explicitly refer to the Pythagorean theorem, or even to a triangle or hypotenuse. Additional examples are given in Fig. 1.

Although students must learn to choose an appropriate strategy, they are denied the opportunity to do so if every problem in an assignment requires the same strategy. For example, if a lesson on the Pythagorean theorem is followed by a group of problems requiring the Pythagorean theorem, students know the appropriate strategy *before* they read each problem. The grouping of problems by strategies is termed *blocked practice*, and the large majority of practice problems in most mathematics textbooks are blocked. Blocked practice served as the control in the study reported here.

In an alternative approach that is evaluated in the present study, a majority of the problems within each assignment are drawn from previous lessons, so that no two consecutive problems require the same strategy—a technique known as *interleaved practice*. With this approach, students must choose an appropriate strategy and not *only* execute it, just as they must choose an appropriate strategy when they encounter a problem during a cumulative exam or high-stakes test. Put another way, blocked practice provides a crutch that might be optimal when students first encounter a new skill, but only interleaved practice allows students to practice what they are expected to know. To create assignments with interleaved practice, the problems within a set of blocked assignments can be rearranged (Fig. 2).

In addition to providing opportunities to practice choosing a strategy, interleaved mathematics assignments guarantee that problems of the same kind are distributed, or *spaced*, across different assignments (Fig. 2). Spacing typically improves performance on delayed tests of learning (e.g., for recent reviews, see Dunlosky et al., 2013; Roediger & Pyc, 2012), and several studies have shown that spacing can improve the learning of mathematics, in particular (Rohrer & Taylor, 2006, 2007; Yazdani & Zebrowski, 2006). To summarize thus far, interleaved practice has two critical features: Problems of different kinds are interleaved (which requires students to choose a strategy), and problems of the same kind are spaced (which usually improves retention).

Previous studies of interleaved practice

Four previously published studies compared the effects of interleaved and blocked mathematics practice (Le Blanc & Simon, 2008; Mayfield & Chase, 2002; Rohrer & Taylor, 2007; Taylor & Rohrer, 2010). In each of the studies, participants received interleaved or blocked practice of different kinds of problems, and interleaving produced better scores on a delayed test. However, in each of these studies, the different kinds of problems (and the corresponding strategies) were nearly identical in appearance (Fig. 3). In one study, for example, every problem included a variable raised to an exponent, and, in another, every problem referred to a prism. We refer to problems with shared features as *superficially similar* problems, and this similarity might hinder students' ability to distinguish or *discriminate* between different kinds of problems. Indeed, the benefit of interleaved practice is often attributed to improved discrimination, as we will detail in the Discussion section. Therefore, the superficial similarity of the problems used in previous studies leaves open the possibility that the test benefit of interleaving is limited to scenarios in which students learn to solve kinds of problems that look alike, and such a boundary condition would curtail the utility of interleaved practice in the classroom, where students encounter problems that are often easily distinguished from other kinds of problems.

Present study

We compared interleaved and blocked mathematics practice in a classroom-based experiment with a counterbalanced, crossover design. Students learned to solve different kinds of problems drawn from their mathematics course, and they received the lessons and assignments from their regular teacher over a period of nine weeks. Two weeks after the last assignment, students sat for an unannounced test. Unlike previous studies of interleaved mathematics practice, the different kinds of problems were superficially dissimilar.

	Problem	1. Choose Strategy	2. Execute Strategy
A	A bug flies 48 m east and then 14 m north. How far is the bug from where it started?	Pythagorean Theorem	$\sqrt{48^2 + 14^2} = 50$
B	A bug flies 48 m east and then 14 m west. How far is the bug from where it started?	Number line arithmetic	$48 - 14 = 34$
C	Find the length of the line segment with endpoints (1, 1) and (5, 4).	Pythagorean Theorem	$\sqrt{3^2 + 4^2} = 5$
D	Find the slope of the line that passes through the points (1, 1) and (5, 4).	$\text{slope} = \frac{\text{rise}}{\text{run}}$	$\frac{4-1}{5-1} = \frac{3}{4}$

Fig. 1 The two steps in the solution of a problem. To solve a problem, students must choose a strategy and then execute it. Superficially similar problems may require different strategies (A and B, or C and D), and superficially dissimilar problems may require the same strategy (A and C). Regardless of similarity, students know the strategy in advance when working a block of problems requiring the same strategy

		Lesson								
		50	51	52	53	54	55	60	70	90
4 problems on the current lesson	1	**50**	51	52	53	54	55	60	70	90
	2	**50**	51	52	53	54	55	60	70	90
	3	**50**	51	52	53	54	55	60	70	90
	4	**50**	51	52	53	54	55	60	70	90
1 problem on each of 8 previous lessons	5	49	**50**	51	52	53	54	59	69	89
	6	48	49	**50**	51	52	53	58	68	88
	7	47	48	49	**50**	51	52	57	67	87
	8	46	47	48	49	**50**	51	56	66	86
	9	45	46	47	48	49	**50**	55	65	85
	10	40	41	46	47	48	49	**50**	60	84
	11	30	31	32	33	34	35	40	**50**	70
	12	10	11	12	13	14	15	20	30	**50**

Fig. 2 A hypothetical set of assignments providing interleaved practice. Each column represents an assignment, and each table entry indicates the lesson number on which the problem is based. For example, if Lesson 50 is on ratios, the corresponding assignment includes four ratio problems and one problem on each of eight lessons seen earlier in the school year (or during the previous school year). Another eight ratio problems (Lesson 50) are distributed across future assignments, with decreasing frequency. In other words, problems of different kinds are interleaved (which requires students to choose a strategy), and problems of the same kind are spaced (which improves retention). Note that the arrangement shown here is not the one that was used in the present study

Method

Participants

The study took place at a public middle school in Tampa, Florida. Three teachers and eight of their seventh-grade mathematics classes participated. Each teacher taught two or three of the classes. Of the 175 students in the classes, 157 students participated in the study. Of these, 140 students attended class on the day of the unannounced test, and only these students' data were analyzed. Nearly all of the students were 12 years of age at the beginning of the school year.

Material

Students learned to solve four kinds of problems drawn from their course (Fig. 4). To confirm that students could not solve these kinds of problems before the experiment, we administered a pretest with one of each kind of problem. Averaged across problems, just 0.7 % of the students supplied both the correct answer (e.g., $x = 7$) and the correct solution (the steps leading to the answer). When scored solely on the basis of answers (which presumably included guesses), the mean score was 3.2 %.

The four kinds of problems were not only superficially different from each other, but also quite unlike other kinds of problems that the students had seen prior to the completion of the experiment. For example, although students ultimately learn how to solve many kinds of equations, a linear equation was the only kind of equation that these students had encountered previously in school (Fig. 4A). Likewise, a linear equation was the only kind of equation that the students had previously graphed (Fig. 4C). The slope problem (Fig. 4D) was also moderately unique, because the term "slope" is used only in limited contexts. However, the proportion word problem (Fig. 4B) does resemble other kinds of word problems.

Design

For the study, we used a counterbalanced crossover design. We randomly divided the eight classes into two groups of four, with the constraint that each group included at least one of the classes taught by each teacher. One group interleaved their practice of problems kinds A and B and blocked their practice of kinds C and D, and the other group did the reverse.

Procedure

During the nine-week practice phase, students received ten assignments with 12 problems each. Across all assignments, the students saw 12 problems of each of the four kinds (Fig. 4). The remaining problems were based on entirely different topics. Students received the ten assignments on Days 1, 15, 24, 30/31, 36, 37, 57, 58, 60, and 64. Every student received the same problems, but we rearranged the problems to create two versions of each assignment—one for each group. The first four problems of kinds A, B, C, and D were the first four problems of Assignments 1, 2, 4, and 5, respectively. If a problem kind was learned by blocked practice, the remaining eight problems appeared in the same assignment as the first four, meaning that the assignment included one block of 12 problems. If a problem

	Problem	1. Choose Strategy	2. Execute Strategy
A	Simplify. $8x^5 \cdot 4x^2$	Add exponents	$32x^{5+2} = 32x^7$
	Simplify. $\frac{8x^5}{4x^2}$	Subtract exponents	$2x^{5-2} = 2x^3$
	Simplify. $(2x^5)^2$	Multiply exponents	$2^2x^{5\cdot 2} = 4x^{10}$
B	Find the volume of a wedge with radius 2 and height 3.	$\frac{1}{2}\pi r^2h$	$\frac{1}{2}\pi 2^2 3 = 6\pi$
	Find the volume of a spheroid with radius 2 and height 3.	$\frac{4}{3}\pi r^2h$	$\frac{4}{3}\pi 2^2 3 = 16\pi$
	Find the volume of a spherical cone with radius 2 and height 3.	$\frac{2}{3}\pi r^2h$	$\frac{2}{3}\pi 2^2 3 = 8\pi$
C	The base of a prism has 5 sides. How many faces does the prism have?	base sides + 2	5 + 2 = 7
	The base of a prism has 5 sides. How many corners does the prism have?	base sides x 2	5 x 2 = 10
	The base of a prism has 5 sides. How many edges does the prism have?	base sides x 3	5 x 3 = 15

Fig. 3 Problems learned in previous studies of interleaved mathematics: Students learned to solve several kinds of problems relating to (A) exponent rules (Mayfield & Chase, 2002), (B) the volume of obscure solids (Le Blanc & Simon, 2008; Rohrer & Taylor, 2007) or (C) prisms (Taylor & Rohrer, 2010) [EE2] In each study, the different kinds of problems (as well as the corresponding strategies) were nearly identical. Note that each of the studies included four or five kinds of problems, but only three are shown here

kind was learned by interleaved practice, the remaining eight problems of the same kind were distributed across the remaining assignments. This meant that students saw the last problem of each kind on a *later* date in the interleaved condition than in the blocked condition, which is an intrinsic feature of assignments with interleaved practice (Fig. 2). The effect of this difference in "true test delay" is detailed in the Results.

Shortly before the scheduled date of each assignment, teachers received paper copies for their students and a slide presentation with solved examples and solutions to each problem. We asked teachers to present the examples before distributing the assignment. On the following school day, teachers presented the solution to each problem while encouraging students to make any necessary corrections to their own solutions. Teachers then collected the assignments. Within two days, one or more of the authors visited the school, scored each assignment (without marking it), and returned the assignments to the teachers. Although these scores do not measure students' mastery, because students could correct their errors while the teacher presented the correct solutions, this scoring of the assignments provided us with evidence of teacher compliance with the experimental procedures.

Students were tested two weeks after the last assignment. We asked teachers not to inform students of the test in advance, because we did not want the final test to be affected by cramming just prior to the test. Teachers did not see the test before it was administered. The students were tested during their regular class, and the teacher and one author proctored each test.

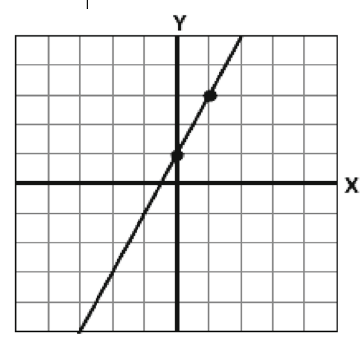
	Problem	1. Choose Strategy	2. Execute Strategy
A	Solve the equation. $3(x + 1) = x + 17$	Isolate *x* terms on one side of the equation	$3x + 3 = x + 17$ $2x + 3 = 17$ $2x = 14$ $x = 7$
B	Penelope's new tractor requires 14 gallons of gas to plow 6 acres. How many gallons of gas will she need to plow 21 acres?	Create a proportion	$\frac{14}{6} = \frac{x}{21}$ $6x = 14 \cdot 21$ $x = 49$
C	Graph the equation. $y = 2x + 1$	Choose at least two values of *x* and find the corresponding values of *y*.	*x* = 0, *y* = 1; *x* = 1, *y* = 3 
D	Find the slope of the line that passes through the points (3, 5) and (6, 7).	$\text{slope} = \frac{\text{rise}}{\text{run}}$	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{6 - 3} = \frac{2}{3}$

Fig. 4 Examples of the four kinds of problems used in the present studies. (A) Solve a linear equation requiring four steps. (B) Solve a word problem using a proportion. (C) Graph an equation of the form, $y = mx + b$, where m and b are integers. (D) Determine the slope of the line defined by two given points with integer coordinates

All of the test problems were novel. The test included three problems of each of the four kinds, and each of the four pages included a block of three problems of the same kind. We created three versions by reordering problems within each block, and students in adjacent chairs received different versions. Students were allotted 36 min and allowed to use their school-supplied basic calculator. Each test was scored on site that day by two raters who were blind to each student's group assignment. The two raters scored each answer as correct or not correct and later resolved the few discrepancies (17 of 1,680). Test score reliability was moderately good (Cronbach's alpha = .78).

Results

A repeated measures comparison of the two halves of the test showed that interleaved practice was nearly twice as effective as blocked practice, $t(139) = 10.49$, $p < .001$ (Table 1). The effect size was large, $d = 1.05$, 95 % CI = [0.80, 1.30]. This benefit of interleaving was observed for each of the four kinds of problems, *p*s < .01. The effect sizes for the four kinds (A, B, C, and D) exhibited a positive trend (0.72, 0.45, 1.00, and 1.27, respectively). This means that the interleaving benefit was *larger* for problem kinds introduced *later* in the practice phase. In other

Table 1 Proportions correct on test

	Mean	*SD*
Interleaved practice	.72	.30
Blocked practice	.38	.35

words, although the true test delay (the interval between the last practice problem and the test) was larger in the blocked condition than in the interleaved condition (see the Procedure section), the problem kinds with larger test delay differences (i.e., that were seen earlier in the practice phase) were associated with *smaller* effect sizes. Although this negative association might reflect order effects—that is, all participants saw the four problem kinds in the same order—we cannot think of a reason why order would matter. In brief, the effect sizes for problem kinds introduced later in the practice phase were larger than the effects for the earlier ones, and this trend was in the opposite direction from what would be expected if the difference in test delays contributed to the observed effect. Furthermore, if this difference did play a role, it might be seen not as a confound, but as an intrinsic feature of interleaved assignments (Fig. 2).

Discussion

Whereas previous studies of interleaved mathematics practice had required students to learn kinds of problems that were nearly identical in appearance (Fig. 3), the results reported here demonstrate that this benefit also holds for problems that do not look alike (Fig. 4). That is, the benefit of interleaved mathematics practice is not limited to the ecologically invalid scenario in which students encounter only superficially similar kinds of problems. Although it might seem surprising that a mere reordering of problems can nearly double test scores, it must be remembered that interleaving alters the pedagogical demand of a mathematics problem. As was detailed in the introduction, interleaved practice requires that students choose an appropriate strategy for each problem and not only execute the strategy, whereas blocked practice allows students to safely assume that each problem will require the same strategy as the previous problem.

However, the interleaved practice effect observed here might reflect the benefit of spaced practice rather than the benefit of interleaving per se. As we explained in the introduction, the creation of interleaved mathematics assignments guarantees not only that problems of different kinds will be interleaved, but also that problems of the same kind will be spaced across assignments, and spacing ordinarily has large, robust effects on delayed tests of retention. We therefore believe that spacing contributed to the large effect observed here ($d = 1.05$). Still, we have reason to suspect that interleaving, per se, contributed as well. In one previous interleaved mathematics study, students in both the interleaved and blocked conditions relied on spaced practice to the same degree, and interleaving nevertheless produced a large positive effect ($d = 1.23$; Taylor & Rohrer, 2010). In the present study, though, we chose to compare interleaved practice to the kinds of assignment used in most textbooks, which is a massed block of problems.

Theoretical accounts of the interleaved mathematics effect

How does interleaving improve mathematics learning? The standard account holds that the interleaving of different kinds of mathematics problems improves students' ability to distinguish or *discriminate* between different kinds of problems (e.g., Rohrer, 2012). Put another way, each kind of problem is a category, and students are better able to identify the category to which a problem belongs if consecutive problems belong to different categories. This ability to discriminate is a critical skill, because students cannot learn to pair a particular kind of problem with an appropriate strategy unless they can first distinguish that kind of problem from other kinds, just as Spanish-language learners cannot learn the pairs PERRO–DOG and PERO–BUT unless they can discriminate between PERRO and PERO.

This discriminability account parsimoniously explains the interleaving effects observed in previous mathematics interleaving studies, because participants in these studies were required to discriminate between nearly identical kinds of problems (Fig. 3). For instance, one of these previous studies included an error analysis, and it showed that the majority of test errors in the blocked condition, but not in the interleaved condition, occurred because students chose a strategy corresponding to one of the *other* kinds of problems that they had learned—for example, using the formula for prism *edges* rather than the formula for prism *faces* (Taylor & Rohrer, 2010). Furthermore, the students in this study were given a second final test in which they were given the appropriate strategy for each test problem and asked only to execute the strategy, and the scores on this test were near ceiling in both conditions. In sum, the data from this earlier experiment are consistent with the possibility that interleaving improves students' ability to discriminate one kind of problem from another (or discriminate one kind of strategy from another).

However, in the present study, discrimination errors appeared to be rare. In a post-hoc error analysis, three raters (two of the authors and a research assistant, all blind to conditions) examined the written solution accompanying each incorrect answer and could not find any solutions in which students "used the wrong strategy but one that solves another kind of problem." The raters then expanded the definition of discrimination error to include solutions with at least one step of a strategy that *might* be used to solve any kind of problem other than the kind of problem that the student should have solved. With this lowered threshold, discrimination errors still accounted for only 33 of the 756 incorrect answers (4.4 %), with no reliable difference between conditions (5.1 % for interleaved, 4.0 % for blocked). For the other incorrect answers, students chose the correct strategy but incorrectly executed it (45.9 %), or they relied on a strategy we could not decipher, often because they did not show their work (49.7 %). The virtual absence of discrimination errors is arguably not surprising, partly because the different kinds of problems did not look alike, and partly because some strategies were obviously an inappropriate choice for some kinds of problems (e.g., trying to graph a line by creating a proportion). The rarity of discrimination errors in the present study raises the possibility that improved discrimination cannot by itself explain the benefits of interleaved mathematics practice.

We suggest that, aside from improved discrimination, interleaving might strengthen the association between a particular kind of problem and its corresponding strategy. In other words, solving a mathematics problem requires students not only to *discriminate* between different kinds of problems, but also to *associate* each kind of problem with an appropriate strategy, and interleaving might improve both skills (Fig. 5). In the present study, for example, students were asked to learn to distinguish a slope problem from a graph problem (a seemingly trivial discrimination) and to associate each kind of problem with an appropriate strategy (e.g., for a slope problem, use the strategy "slope = rise/run"), and the latter skill might have benefited from interleaved practice. Yet why would interleaving, more so than blocking, strengthen the association between a problem and an appropriate strategy? One possibility is that blocked assignments often allow students to ignore the features of a problem that indicate which strategy is appropriate, which precludes the learning of the association between the problem and the strategy. In the present study, for example, students who worked 12 slope problems in immediate succession (i.e., used blocked practice) could solve the problems without noticing the feature of the problem (the word "slope") that indicated the appropriate strategy (slope = rise/run). In other words, these students could repeatedly execute the strategy $(y_2 - y_1)/(x_2 - x_1)$ without any awareness that they were solving problems related to slope. In brief, blocked practice allowed students to focus only on the execution of the strategy, without having to associate the problem with its strategy, much like a Spanish-language learner who misguidedly attempts to learn the association between PERRO and DOG by repeatedly writing DOG.

It might be possible to experimentally tease apart the effects of interleaving on discrimination and association. In one such experiment, participants would receive either blocked or interleaved mathematics practice during the learning phase, as they typically do, and then take two tests. The first test would assess only discrimination. For example, students might be shown a random mixture of five problems—four problems of one kind (e.g., word problems requiring a proportion) and one problem of a different kind (e.g., a word problem requiring the Pythagorean theorem)—and then be asked to identify the problem that does not fit with the others (the Pythagorean theorem problem). Students would repeat this task many times with different kinds of problems. On a second test measuring both discrimination and association, students would see problems one a time and, for each problem, choose the correct strategy, but not execute it. Scores on the first test (discrimination only) should be greater than scores on the more challenging second test (discrimination and association), with larger differences between the two test scores reflecting a poorer ability to associate a kind of problem and its strategy. Therefore, *if* interleaving improves association, the difference between the two test scores should be smaller for students who interleaved rather than blocked.

Category learning

Finally, although we focused here on mathematics learning, several studies have examined the effect of interleaved practice on category learning. For example, participants might see

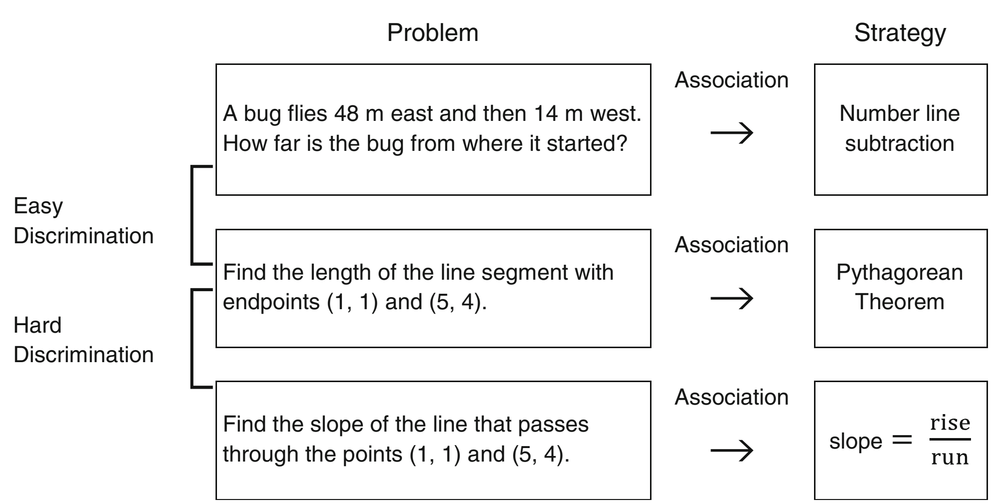


Fig. 5 Discrimination and association. The solution of a mathematics problem requires that students discriminate one kind of problem from another and associate each kind of problem with an appropriate strategy. Interleaving might improve both skills

photographs of different kinds of birds (jays, finches, swallow, etc.) one at a time, in an order that was either blocked (each of the jays, then each of the finches, etc.) or interleaved (jay, finch, swallow, etc.), and interleaving would produce greater scores on a subsequent test requiring participants to identify previously unseen birds (e.g., Birnbaum, Kornell, Bjork, & Bjork, 2013; Kang & Pashler, 2012; Kornell & Bjork, 2008; Wahlheim, Dunlosky, & Jacoby, 2011; but see Carpenter & Mueller, 2013). As with the results of previous interleaved mathematics tasks, the positive effect of interleaving on category learning could also be attributed to an improved ability to discriminate between, say, a jay and a finch. To our knowledge, though, it remains an untested possibility that this effect might also reflect a strengthened association between each category (e.g., finches) and the category name ("finch"). The relative contributions of enhanced discrimination and stronger associations to interleaving effects could be disentangled by an experiment analogous to the mathematics experiment proposed in the previous section: Participants would receive two tests: a discrimination-only test requiring them to sort birds (or identify the one bird that is different from others), and the usual test requiring them to name novel birds, which would require both discrimination and association. In summary, although strong evidence exists showing that interleaved practice can improve both mathematics learning and category learning, it seems unclear why either of these effects occur.

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CRAAP TEST

When evaluating sources you must ensure that they meet the following criteria before using them in your scholarly work.

- Currency—is the article current? Currency is usually thought of as in the past five years, however, some seminal pieces of research or books written 15 or more years ago may still be current. Standards and regulations change more slowly, so these topics can have a longer period of currency. Something in fields like biomed or artificial intelligence etc., is probably measured in much shorter time periods.
- Relevance—is the content of the source relevant to the question or subject being considered?
- Authority—does the author (or authors) have authority in the subject? Although Steve Jobs may have been an amazing designer, he probably had little authority when it came to the best way to raise puppies.
- Accuracy—how accurate do you judge the information to be? Information supplied by most government sites is usually more accurate than information supplied by a marketing site. Information provided by a standards organization such as Underwriters Laboratory is usually more accurate than information provided by a blogger.
- Purpose—For what purpose was the information produced? If the article or information was produced to persuade readers or to convey information, the content may be different. If the purpose is aligned with the person using the information, it is probably more reliable or useful.

PUTTING WORDS IN THE CORRECT ORDER

This worksheet is designed to build your skills in putting words in the correct order.

EXERCISE

Put the following words into the correct order.

For example: drink/the/performance/a/You/after/buy/can becomes:

"You can buy a drink after the performance."

1. The/is/music/thing/the/about/love/film/I/that/the
2. order/It/realize/is/word/the/important/to/correct
3. We/have/about/it/must/it/before/forgotten/seen/and
4. ordinary/accidents/year/items/by/Every/of/are/thousands/caused
5. building/woman/furry/A/from/a/was/dog/burning/her/by/rescued
6. animals/business/It/to/after/an/exotic/look/is/expensive
7. nervous/students/Taking/very/time/an/test/a/for/be/English/can

STUDENT ANSWERS

1.
2.
3.
4.
5.
6.
7.

PUTTING WORDS IN THE CORRECT ORDER

This worksheet is designed to build your skills in putting words in the correct order.

ANSWERS

1. The thing that I love about the film is the music.
2. It is important to realize the correct word order.
3. We must have seen it before and forgotten about it.
4. Every year thousands of accidents are caused by ordinary items.
5. A woman was rescued from a burning building by her furry dog.
6. It is an expensive business to look after exotic animals.
7. Taking an English test can be a very nervous time for students.